Resistive Wall Tearing Mode Locking

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PPPL Disruption Theory Workshop 2015

RWTM Locking

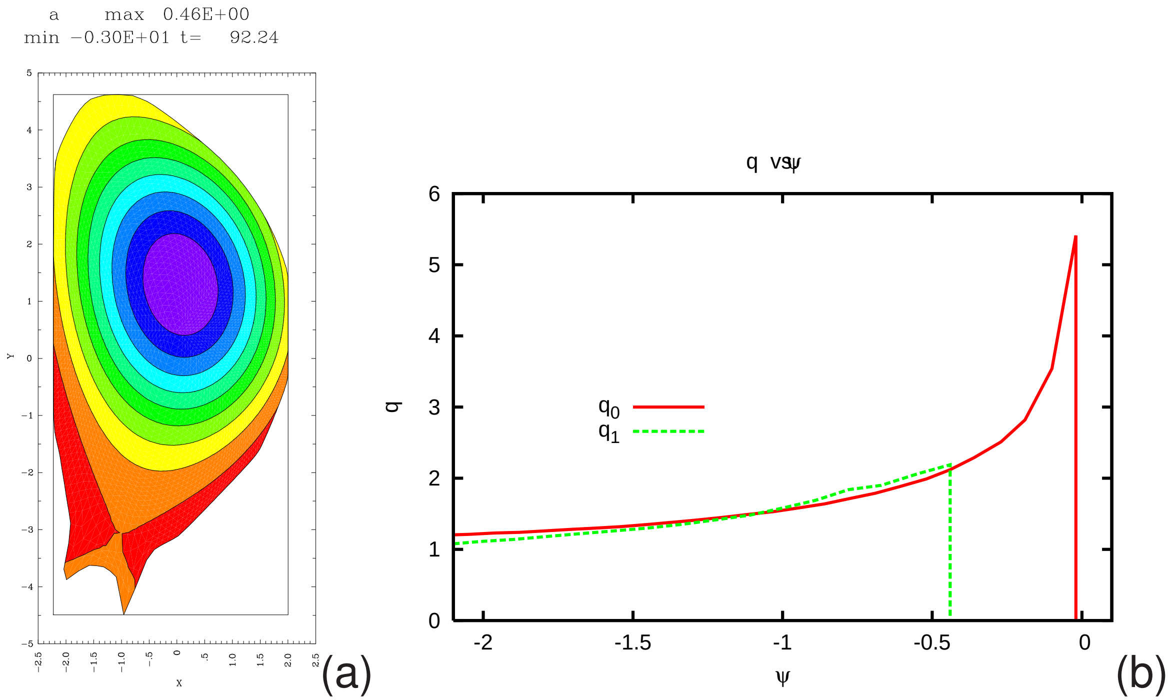
- The most common type of disruption involves a saturated, rotating $(2,1)$ magnetic island, near the edge of the plasma.
- The rotation slows down and locks to the wall, and the plasma disrupts.
  - why does the rotating tearing mode saturate without disrupting?
  - why does the mode cause a disruption when locked?
  - what causes locking?
- resistive wall has an essential role
- will consider resistive wall tearing modes (RWTMs)

Outline

- Linear RWTM in asymmetric vertical displacement events (AVDEs)
- Nonlinear RWTM
  - effect of wall resistive penetration time τ_{wall}
  - shear free rotation and effective τ_{wall}
  - effect of sheared rotation
- sheared rotation and locking

Scrape Off of Flux by VDE

$(2, 1)$ modes near the plasma edge were studied in the context of AVDEs [H. Strauss, R. Paccagnella, J. Breslau, L. Sugiyama, S. Jardin, Sideways Wall Force Produced During Tokamak Disruptions, Nucl. Fusion **53**, 073018 (2013).]

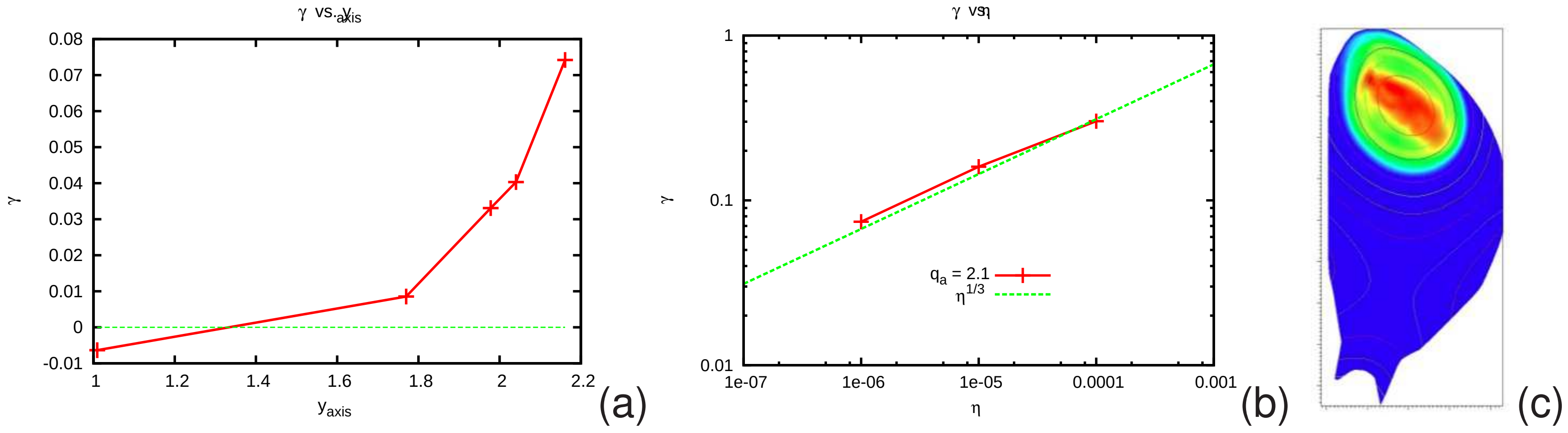


(a) (b)

(a) The poloidal magnetic flux ψ is shown when the initial separatrix contour reaches the wall during a VDE.

(b) q profiles corresponding the initial state and (a). The vertical lines are drawn at the last closed flux surface.

AVDE linear growth rate and nonlinear structure



(a) linear growth rate γ vs. vertical displacement y_{axis}. As y_{axis} increases, $q \to 2$.

(b) For $q_a \gtrsim 2, \gamma \propto S^{-1/3}$. For large τ_{wall} the growth rate is independent of τ_{wall}, but for for smaller τ_{wall}, the growth rate decreases with τ_{wall}.

(b) pressure in a nonlinear evolution. The mode structure is predominantly $(m, n) = (2, 1)$. The mode was not rotating before the disruption.

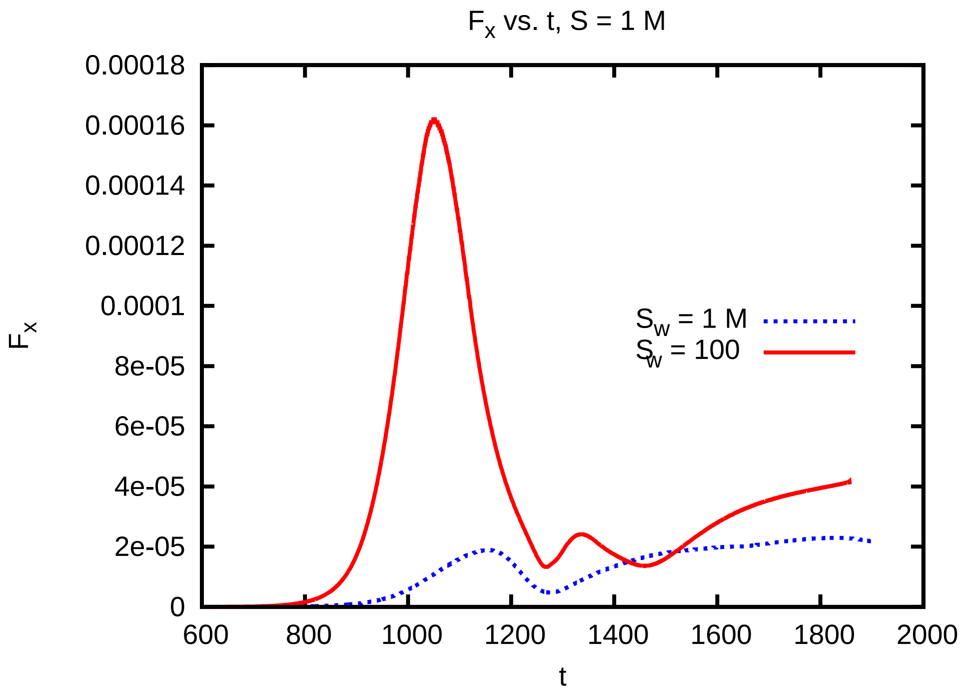
RWTM - effect of τ_{wall}, no rotation, no VDE

Initial state was prepared by setting current and pressure zero for $\psi < \psi_a$, where $q(\psi_a) = q_a$, and $q_a \approx 2.1$

Nonlinear simulation were done, with $S = 10^6$, and τ_{wall}/τ_A (a)$= 10^6$, (b) $= 100$.

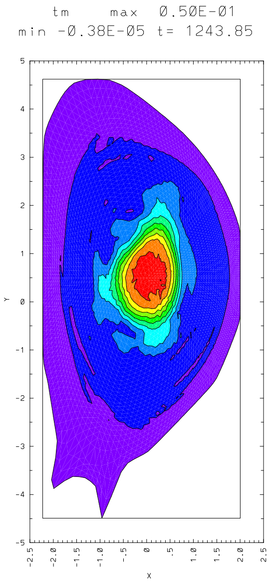
For case (a) F_x saturates at low amplitude. The wall is like an ideal wall.

For case (b) F_x reaches a much larger value. The wall is like no - wall.

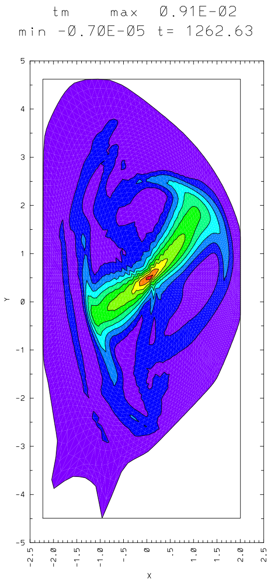


Wall force amplitude is an order larger for resistive wall.

RWTM - effect of τ_{wall}, no rotation

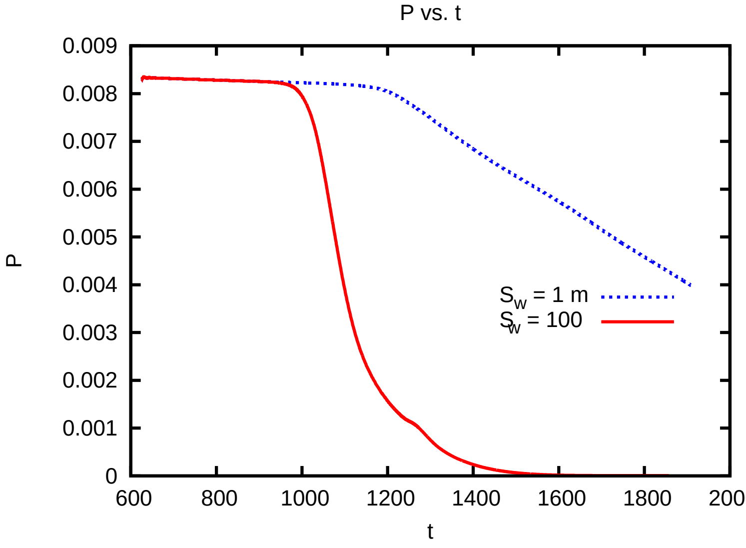


(a)

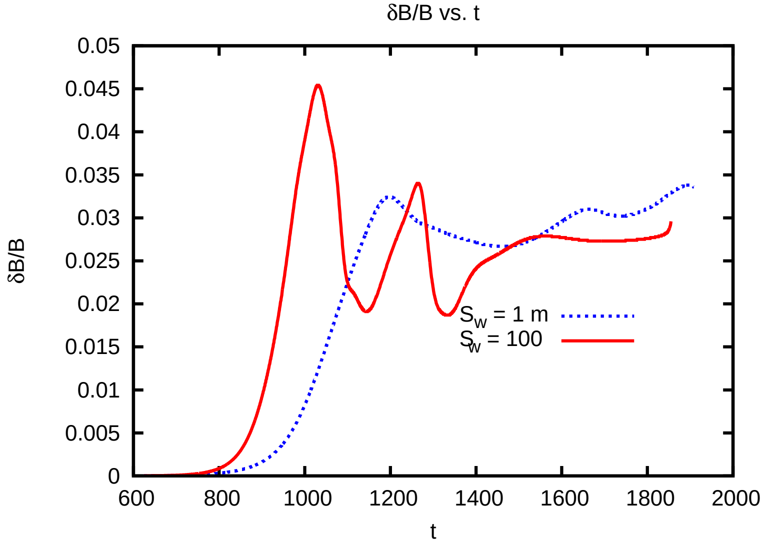


(b)

(a) pressure, $t = 1243\tau_A$, $S = 10^6, S_w = 10^6 = \tau_w/\tau_A$
(b) pressure, $t = 1262\tau_A$, $S = 10^6, S_w = 100$,
(c) P(t): (a) pressure loss and (b) fast TQ.
(d) $\delta B/B$ at the wall for (a) and (b).



(c)



(d)

RWTM - effect of resistive wall on nonlinear stability

Assume nonlinear saturation related to Δ', a measure of free energy

$$\Delta' = \frac{\psi'_+}{\psi} - \frac{\psi'_-}{\psi} \tag{1}$$

at rational $(2,1)$ surface r_s, with wall at $r_w > r_s$.

ideal wall: $\psi(r_w) = 0$.

$$\frac{\psi'_+}{\psi} = -\frac{2}{r_s}\frac{r_w^4 + r_s^4}{r_w^4 - r_s^4} \tag{2}$$

no wall: $\psi = r_s^2/r^2$,

$$\frac{\psi'_+}{\psi} = -\frac{2}{r_s} \tag{3}$$

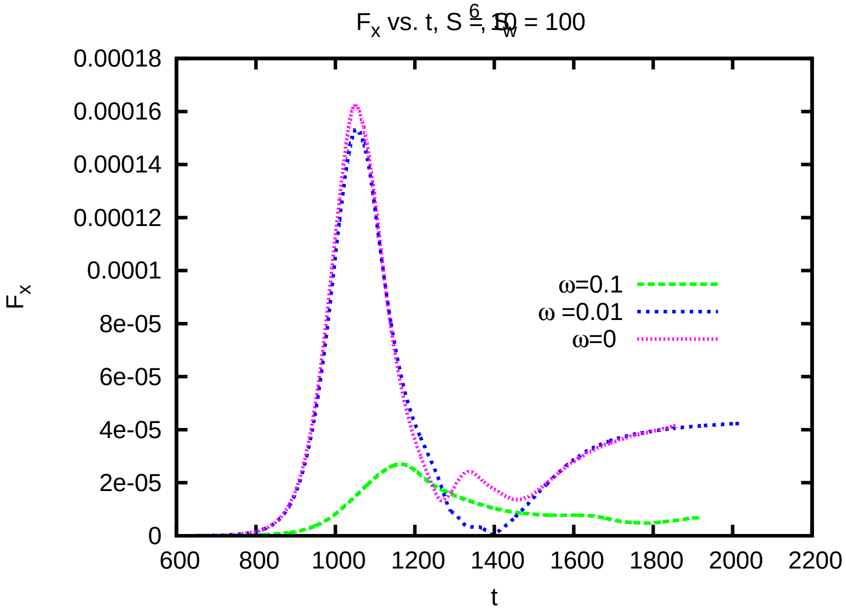
ψ'_-/ψ is the same for both cases, and

$$\frac{r_w^4 + r_s^4}{r_w^4 - r_s^4} > 1 \tag{4}$$

for $r_s > 0$, so that Δ' is larger for no wall than for ideal wall.

RWTM - effect of shear free rotation

Shear free rotation was modeled by going to a toroidally rotating frame, in which the plasma was stationary and the wall rotated.
The simulations were done with $S = 10^6, S_{wall} = 100$, with rotation frequencies: $\omega\tau_A =$ (a) 0, (b) 0.01 (c) 0.1
The slow rotation is like no rotation. Fast rotation is like ideal wall.
Simple theory shows that $\psi \approx 0$ on the wall if $\omega\tau_{wall} \gg 1$.

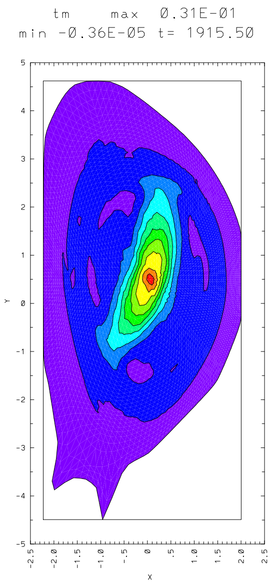


$$\frac{\partial\psi}{\partial t} = \frac{\eta_w}{\delta}(\psi'_{vac} - \psi'_{plas}) \tag{5}$$

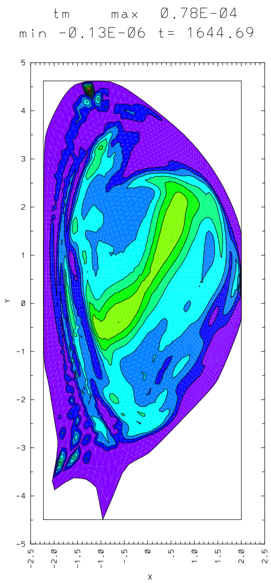
$$\psi'_{vac} = -\frac{m}{r}\psi, \qquad \frac{\partial\psi}{\partial t} = (\gamma + i\omega)\psi, \qquad \tau_w = \frac{\delta r}{\eta_w} \tag{6}$$

$$\psi = -\frac{r\psi'_{plas}}{m + (\gamma + i\omega)\tau_w} \qquad \psi \approx 0, \qquad |\gamma + i\omega|\tau_w \gg m \tag{7}$$

RWTM - effect of shear free rotation



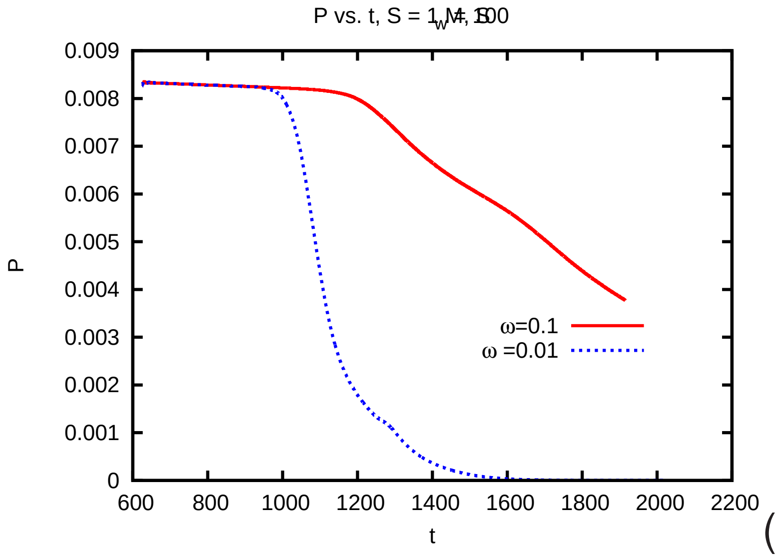
(a)



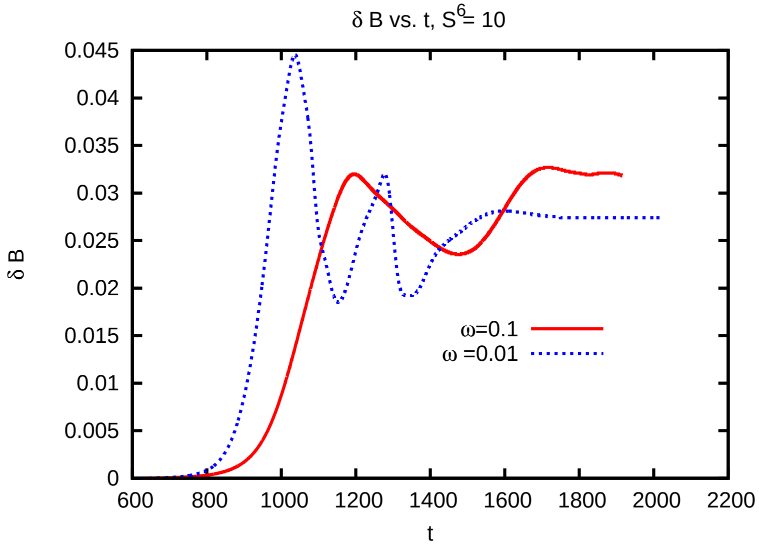
(b)

simulations with $S = 10^6, S_w = \tau_w/\tau_A = 100$

(a) pressure, $t = 1915\tau_A, \omega = 0.1$.
(b) pressure, $t = 1261\tau_A, \omega = 0.01$.
(c) P(t) for (a): similar to $\omega = 0, S_w = 10^6$ and (b) like $\omega = 0, S_w = 100$.
(d) $\delta B/B$ for cases (a) and (b).

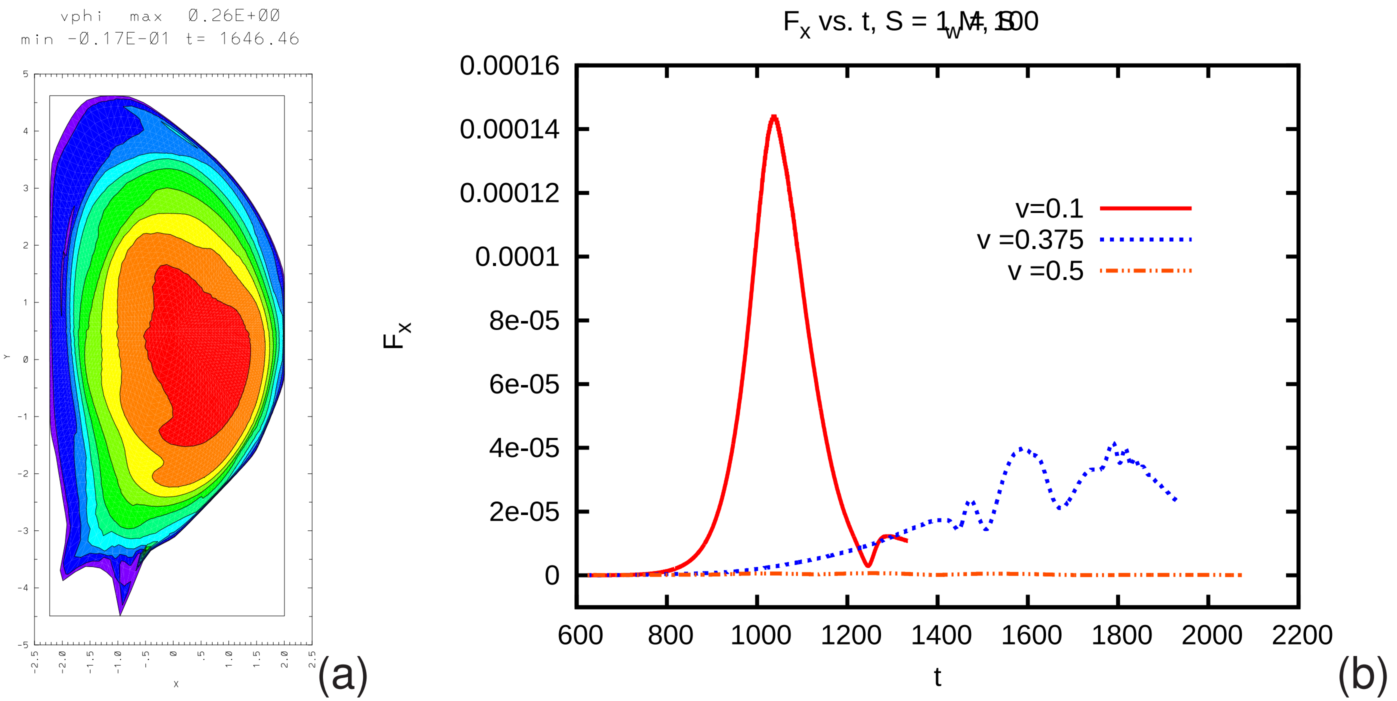


(c)



(d)

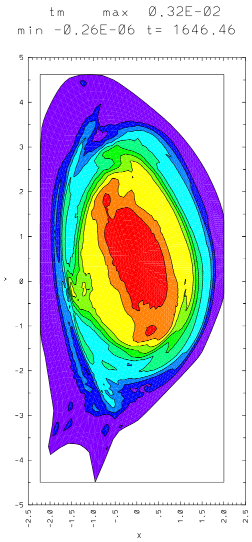
RWTM - sheared rotation



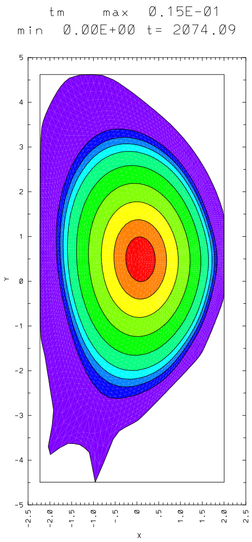
(a) A sheared velocity was introduced, which was zero at the plasma edge, with $S = 10^6, S_w = 100$. From a nonlinear simulation.

(b) The peak velocity was $v_\phi = 0.1, 0.375, 0.5$. For the fastest rotation , the mode is suppressed. Sheared rotation can completely stabilize the mode.

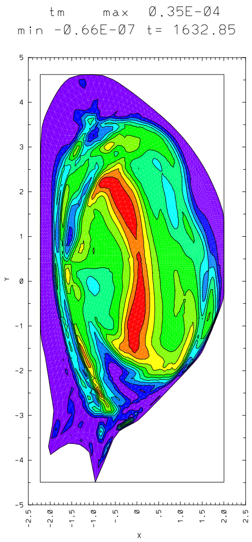
RWTM - effect of sheared rotation



(a)



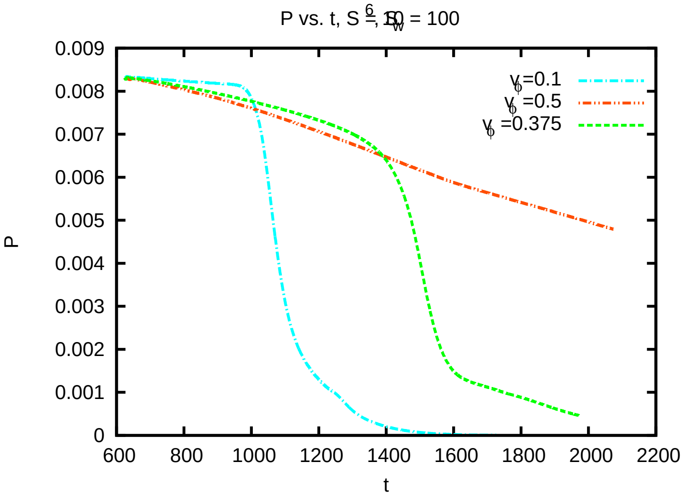
(b)



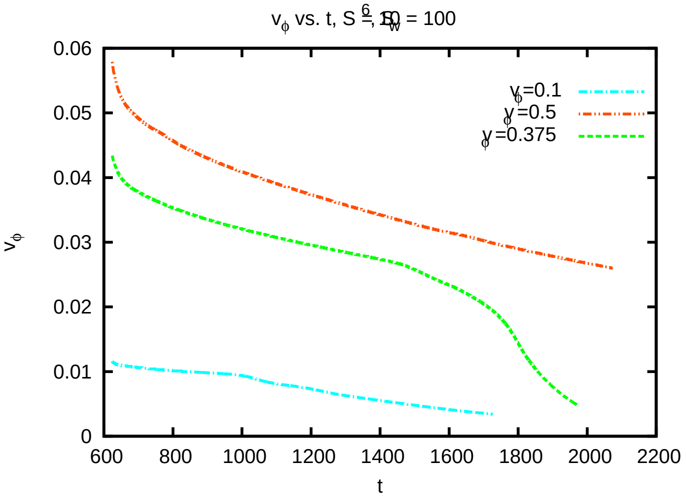
(c)

pressure with $S = 10^6, S_w = 100$.

(a) $t = 1646\tau_A, v_\phi = 0.375$.
(b) $t = 2074\tau_A, v_\phi = 0.5$.
(c) $t = 1632\tau_A, v_\phi = 0.1$.
(d) $P(t)$ for $v_\phi = 0.1, 0.375, 0.5$
(e) v_ϕ for $v_\phi = 0.1, 0.375, 0.5$



(d)



(e)

Self Locking

$$\rho \frac{dv_{\|}}{dt} = -\mathbf{b} \cdot \nabla p \tag{8}$$

$$\gamma p = -\frac{2}{3} p \mathbf{b} \cdot \nabla v_{\|}, \qquad p = T\rho \qquad \mathbf{b} \cdot \nabla T \approx 0 \tag{9}$$

$$\frac{d\overline{v}_{\|}}{dt} = \nabla \cdot \kappa_{eff} b_r^2 \nabla \overline{v}_{\|}, \qquad \kappa_{eff} = \frac{2p}{3\gamma\rho} \tag{10}$$

similarly,

$$\frac{d\overline{p}}{dt} = \frac{1}{r}\frac{d}{dr} r \kappa_{\|} b_r^2 \frac{d\overline{p}}{dr} \tag{11}$$

$$\frac{d}{dt}\int v_{\|} dR dZ = \oint \left(\kappa_{eff} b_r^2 \frac{dv_{\|}}{dr} + \frac{v_A^2}{4\pi} \overline{b}_r b_\theta^2 \right) d\ell \tag{12}$$

Net loss of $v_{\|}$ requires b_r non zero at the wall. Last term is drive requiring asymmetry in θ. [H. Strauss, L. Sugiyama, R. Paccagnella, J. Breslau, S. Jardin, Tokamak Toroidal Rotation caused by AVDEs and ELMs, Nuclear Fusion **54**, 043017 (2014)]. Ballooning could give asymmetry. Steady state possible.

Summary

- RWTMs occur in AVDEs
  - flux scrape off causes $q_a \to 2$ at the plasma edge
- Nonlinear RWTM
  - chose a very unstable case with $q_a \gtrsim 2$, will try a slower growing mode
  - no rotation: mode grows larger with a resistive wall than with ideal wall.
    - Δ' is larger.
    - There is pressure loss with an ideal wall, because $B_n \neq 0$.
  - shear free rotating wall
    - fast rotation is like ideal wall
  - sheared rotation
    - sheared rotation is like shear free
    - sheared rotation can also stabilize modes completely
    - locking is more like shear free, with a pre existing mode
  - self locking if $b_n \neq 0$, drive